$$(y^2-1)\,dx + x\,dy = 0$$

IF x IS THE I.V. $\left(\frac{dy}{dx}\right)$

$$(y^2-1) + x\frac{dy}{dx} = 0$$

$$x\frac{dy}{dx} + y^2 = 1 \longrightarrow \text{NON-LINEAR}$$

IF y IS THE I.V. $\left(\frac{dx}{dy}\right)$

$$(y^2-1)\frac{dx}{dy} + x = 0 \longrightarrow \text{LINEAR}$$

DEFINITION:

GIVEN AN ODE
IF WE SUBSTITUTE $\phi(x)$ FOR y
AND THE ODE IS AN IDENTITY
THEN $\phi(x)$ IS AN EXPLICIT SOLUTION OF THE ODE

$\uparrow$

$y=\phi(x)$

CONSIDER ODE $y'' + y = -2\sin x$

IS $\phi(x) = x\cos x$ AN EXPLICIT SOL'N ?

$$\phi'(x) = 1 \cdot \cos x + x \cdot (-\sin x)$$

$$= \cos x - x\sin x$$

$$\phi''(x) = -\sin x + (-1)\sin x + (-x)\cos x$$

$$= -2\sin x - x\cos x$$

$$\phi'' + \phi = -2\sin x \cancel{- x\cos x} \cancel{+ x\cos x}$$

$$= -2\sin x$$

$\phi(x) = x\cos x$ IS AN EXPLICIT SOL'N

CAN'T ALWAYS GET EXPLICIT SOL'N

CONSIDER ODE $(e^y - x\sin y)\frac{dy}{dx} + \cos y = 0$

CONSIDER $\boxed{e^y + x\cos y = 0}$

DIFFERENTIATE BOTH SIDES IMPLICITLY WITH RESPECT TO X

DIFF WRT

$$\frac{d}{dx}\left(e^y + x\cos y\right) = \frac{d}{dx}0$$

$$\frac{d}{dy}e^y \cdot \frac{dy}{dx} + 1\cdot\cos y + x\cdot\frac{d}{dy}\cos y\,\frac{dy}{dx} = 0$$

$$e^y\frac{dy}{dx} + \cos y + x(-\sin y)\frac{dy}{dx} = 0$$

$$(e^y - x\sin y)\frac{dy}{dx} + \cos y = 0$$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

$$\frac{df(y)}{dx} = \frac{df(y)}{dy}\frac{dy}{dx}$$

$e^y + x\cos y = 0$ IS AN IMPLICIT SOL'N OF

$$(e^y - x\sin y)\frac{dy}{dx} + \cos y = 0$$

FIXED CONSTANT

$$\frac{dA}{dt} = -kA\ , \quad k > 0 \qquad \text{ORDER} = 1$$

$A(t) = A_0 e^{-kt}$ IS AN EXPLICIT SOL'N FOR ANY CONSTANT A_0

ARBITRARY CONSTANT

CONSIDER

$$\frac{d^2h}{dt^2} = -g$$

h = HEIGHT OF A FALLING OBJECT AT TIME t

ORDER = 2

$$\frac{dh}{dt} = \int -g\,dt$$

$$= -gt + C_1$$

$$h = \int(-gt + C_1)\,dt$$

$h = -\frac{1}{2}gt^2 + C_1 t + C_2$ IS AN EXPLICIT SOL'N FOR ANY CONSTANTS C_1, C_2

$\frac{dy}{dx} = k$[illegible] $\qquad y = Ck$[illegible] $= C$[illegible]

1.2 INITIAL VALUE PROBLEM (IVP)

DEF'N: AN IVP IS AN n-ORDER ODE WITH VALUES $x_0, y_0, y_1, \ldots, y_{n-1}$

WHERE

$$\left.\begin{aligned}
&y(x_0) = y_0 \\
&\left.\frac{dy}{dx}\right|_{x=x_0} = y_1 \\
&\left.\frac{d^2y}{dx^2}\right|_{x=x_0} = y_2 \\
&\quad\vdots \\
&\left.\frac{d^{n-1}y}{dx^{n-1}}\right|_{x=x_0} = y_{n-1}
\end{aligned}\right\} \text{INITIAL VALUES}$$

CONSIDER THE IVP $y'' + y = -2\sin x$

$$y(0) = 2 \;\checkmark$$

$$y'(0) = 3 \;\checkmark$$

IS $y = 2\cos x + 2\sin x + x\cos x$ A SOL'N OF THE IVP?

$$y(0) = 2\cos 0 + 2\sin 0 + 0\cos 0$$

$$= 2 \;\checkmark$$

$$y' = -2\sin x + 2\cos x + \cos x - x\sin x$$

$$= -2\sin x + 3\cos x - x\sin x$$

$$y'(0) = -2\cdot 0 + 3\cdot 1 - 0\cdot 0$$

$$= 3 \;\checkmark$$

$$y'' =$$

$$y'' + y = \quad \overset{?}{=} -2\sin x$$